

A correlation between the amount of dark matter in elliptical galaxies and their shape.

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We discuss the correlation between the dark matter content of elliptical galaxies and their ellipticities. We then explore a mechanism for which the correlation would emerge naturally. Such mechanism leads to identifying the dark matter particles to gravitons. A similar mechanism is known in Quantum Chromodynamics (QCD) and is essential to our understanding of the mass and structure of baryonic matter.

Introduction The influence of dark matter is ubiquitous in the universe [1]. Let us list but a few cosmological observations which indicate that the presence of dark matter is necessary:

- Dark matter explains why the outskirts of disk galaxies can spin so rapidly [2]. At radii greater than about 10 kpc for typical spiral galaxies, rotational speeds are significantly faster than expected if these galaxies would consist only of baryonic matter bounded by Newtonian gravity.
- Dark matter keeps galaxies confined in clusters even though galactic speeds exceed the liberation speeds expected from the cluster baryonic mass and Newton's Law [3].
- Dark matter is necessary to aggregate baryonic matter from its relatively smooth primordial distribution to the large-scale structures presently observed [4].
- Dark matter solves the abundance problem in the primordial nucleosynthesis of deuterons [5].

Dark matter is important to understand the internal dynamics of galaxies. There are many correlations between the different quantities characterizing a galaxy. The reasons for some of them are understood while others are still phenomenological observations yet to be explained. The most evident characteristics of an elliptical galaxy are its mass M and ellipticity ε . From our present understanding of galaxy formation and galactic dynamics, there is no reason for M and ε to be significantly correlated. However, dark matter phenomenology is still not well understood at galactic scale. It is thus worthwhile to investigate whether the principal properties of elliptical galaxies are correlated.

The correlation between mass and ellipticity Such an investigation was done in Ref. [6], with a significant correlation established. For the study, we have considered only publications reporting the total mass of at least several elliptical galaxies. Various selection criteria were applied to isolate the signal from the background. Only elliptical galaxies of medium masses without peculiarities were retained. In particular, galaxies in significant interaction with other galaxies were rejected, as were giant or dwarf galaxies. The selection criteria were devised to select one type of elliptical galaxy (typical medium size galaxies) in a relaxed state so that its mass estimate is reliable. The criteria were determined before investigating the mass-ellipticity correlation and, as such, are unlikely to have biased the analysis. In all, 685 determinations of total galactic masses were used. With such a large number, the problem of knowing the intrinsic ellipticity of the galaxies (remember that only projected ellipticities can be observed) can be addressed statistically.

Different methods were used in the publications to assess galactic masses. The analyses were based on the virial theorem [7], stellar dynamics modeling [8], interstellar gas X-ray emissions [9], observation of planetary nebulae and globular clusters [10]-[11], observation of gas disks embedded in elliptical galaxies [12] or strong lensing [13]. The references for each given method indicate the publications used in the data mining analysis of Ref. [6]. The independence of the multiple techniques guards the analysis [6] against methodological biases.

The total masses normalized to galactic luminosity, M/L , (or to stellar mass, M/M_*) were plotted versus the ellipticities ε . The relations were then fitted by a straight line for the 41 samples of galaxies given in [7]-[13]. Four examples of such fits out of the 41 are shown in Fig. 1. A non-zero slope, $d(M/L)/d\varepsilon \gtrsim 3\sigma$ (with σ the uncertainty of the slope), signals a correlation for the given sample. The values of $d(M/L)/d\varepsilon$ were then corrected on a statistical basis for the fact that we only observe the projection of the ellipse with an unknown viewing angle. To assess the correction, we assumed that the galaxies are axisymmetric, and thus can be characterized by a single ellipticity, ε_{true} . A Gaussian distribution of ε_{true} was generated. Assuming that galaxies are randomly oriented with respect to Earth, the simulated distribution of projected — or apparent — ellipticities ε_{app} is obtained from the initial distribution of ε_{true} . The parameters of the Gaussian distribution were then adjusted until the simulated distribution of ε_{app} matched the observed one. Setting $M/L = a\varepsilon_{true}$, a was determined by matching the constructed two-dimensional

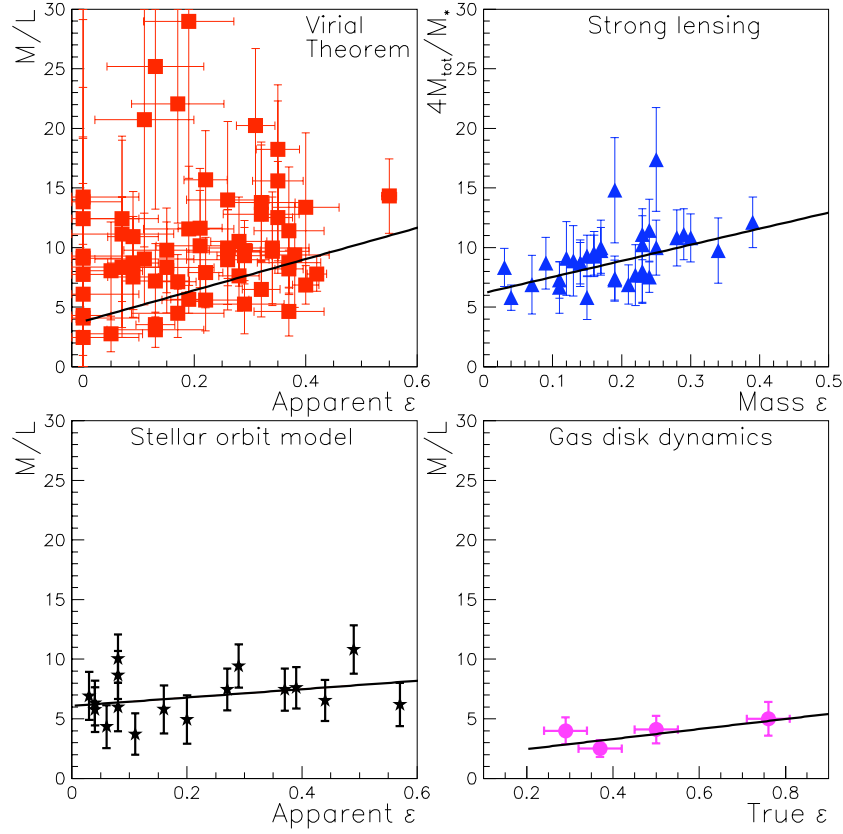


FIG. 1: Galactic mass over luminosity in solar M_{\odot}/L_{\odot} units versus ellipticity from four publications using different methods for assessing galactic masses.

distribution of $a\varepsilon_{true}$ vs ε_{app} to the experimental distribution of M/L vs. ε_{app} . The value of a gives the slope $d(M/L)/d\varepsilon$, corrected for the projection effect. The 41 corrected determinations of $d(M/L)/d\varepsilon$ were combined, accounting for their respective statistical accuracies, their precision, different M/L normalizations, and correlations between mass determinations using the same technique and overlapping sets of galaxies. Fig. 2 shows the 41 corrected determinations of $d(M/L)/d\varepsilon$, the average for each method and the overall average. All averages indicate a positive slope $d(M/L)/d\varepsilon$, although some are compatible with zero. The overall average $d(M/L)/d\varepsilon = (14.59 \pm 3.80) M_{\odot}/L_{\odot}$ is clearly positive. This slope is steep given the average $M/L = 7.7 M_{\odot}/L_{\odot}$.

The correlation between M/L and ε can be physical or it could be a systematic bias. A methodological bias is unlikely since data from independent methods and different authors were used. Other possible biases were investigated. The correlation was re-assessed by using the Pearson correlation coefficient and by fitting the data with a quadratic rather than a linear form. These re-assessments also indicate a strong correlation. A survival analysis was performed in which individual data sets were removed in turn and the correlation re-estimated. The resulting distribution of the $d(M/L)/d\varepsilon$ has a root-mean square width of 0.514, well below the average value of 14.59. The possibility of lenticular (S0) galaxy contamination of the samples was investigated, since S0 and elliptical galaxies are difficult to distinguish at small ε . Since the M/L ratios of S0 galaxies tend to be smaller than those of elliptical galaxies, a contamination would induce an increase of M/L with ε . However, even the upper limit for S0 contamination was found to be too small to explain the observed correlation. The correlation could also originate from the many relations between quantities describing galaxies. This was investigated and no interrelations leading to a $(M/L)-\varepsilon$ correlation could be identified. Thus, at the present state of our knowledge, the correlation seems to be physical rather than originating from a methodological, observational or instrumental bias. In any case, the correlation, regardless of its origin, is an important fact that must be considered in studies of galaxy dynamics.

The average correlation slope $d(M/L)/d\varepsilon = (14.59 \pm 3.80) M_{\odot}/L_{\odot}$ and the average value $M/L = 7.7 M_{\odot}/L_{\odot}$ correspond to the intercept $M/L = (3.3 \pm 1.5) M_{\odot}/L_{\odot}$ at $\varepsilon = 0$. With the normalization used here, corresponding to luminosities obtained in the blue band, $M_*/L \simeq 4 M_{\odot}/L_{\odot}$ when only the stellar mass M_* is counted. Thus, the roundest galaxies, including those considered in [11], contain little dark matter. The correlation is puzzling since there

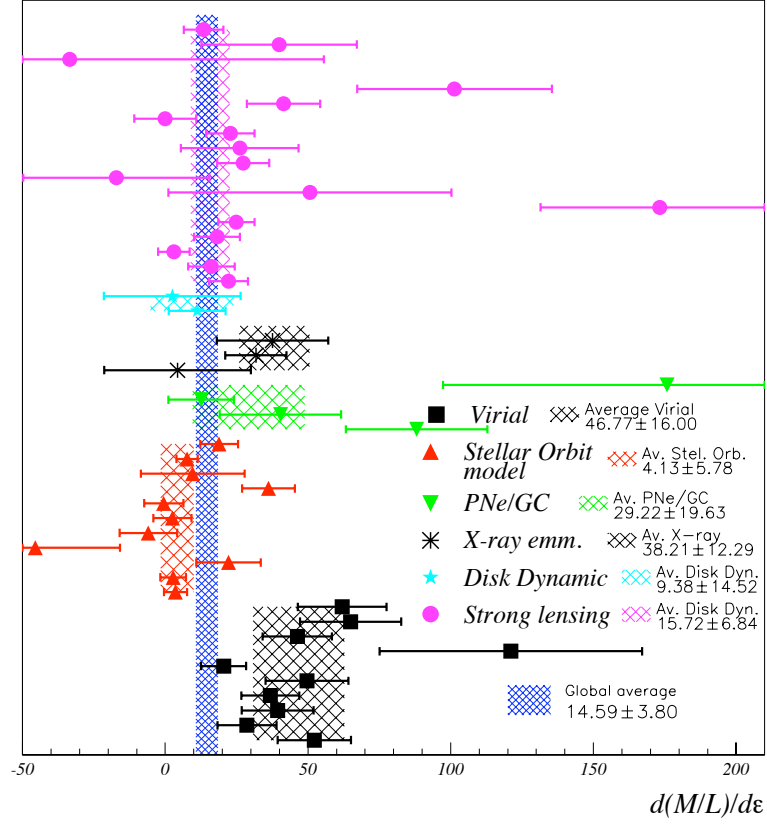


FIG. 2: Slopes $d(M/L)/d\varepsilon$ for the samples from Refs. [7]-[13]. The six different symbols distinguish the methods used to obtain M/L . The corresponding bands indicate their averages. The blue band is the global average of all methods.

is no basic reasons for it in the context of the Cold Dark Matter model and since dark matter is needed to trigger the formations of structures that will latter form elliptical galaxies.

Possible Explanation: Gravitons as dark matter particles Considering gravitons as the particles that constitute dark matter offers a possible solution to these puzzles. (A correlation between M/L and ε was predicted in this context [14].) At galactic distance scales and for the gravity fields involved, the classical formulation of gravity, General Relativity (GR), can be used to investigate the behavior of gravitons and to determine if they can account for dark matter. Consequently, although we will be using the language of quantum field theory and discuss gravitons rather than classical fields, the argument does not depend on a contingent quantization of gravity.

Since gravitons carry energy and momentum, they couple to each other with a coupling \sqrt{G} at the amplitude level, where G is the Newton's constant. This well-known fact is described e.g. in textbooks [15]-[16] and, in classical field language, is responsible for the non-linearity of GR's equations. In natural units ($\hbar = c = 1$), the coupling G is small, e.g. the gravitational potential near a proton is $\frac{GM_p}{r} = 3.8 \times 10^{-38}$, with M_p the proton mass and $r = 8.4 \times 10^{-16}$ m the proton radius [17], to be compared with the force couplings $\alpha \approx 7 \times 10^{-3}$ for the electromagnetic force (QED) and $\alpha_s \approx 1$ for the strong nuclear force (QCD). The extreme smallness of G makes the effects of mutual coupling of gravitons usually negligible. However, for massive enough systems, the effect should become important. For example, for a typical galaxy, $\frac{GM}{r} \simeq 10^{-3}$.

The Lagrangian of GR is $\mathcal{L}_{GR} = \frac{1}{16\pi G} \sqrt{\det(g_{\mu\nu})} g_{\mu\nu} R^{\mu\nu}$, with $g_{\mu\nu}$ the metric tensor and $R_{\mu\nu}$ the Ricci tensor. Expanding \mathcal{L}_{GR} in terms of the tensor gravity field $\varphi_{\mu\nu}$ yields [16]:

$$\mathcal{L}_{GR} = [\partial\varphi\partial\varphi] + \sqrt{16\pi G} [\varphi\partial\varphi\partial\varphi] + 16\pi G [\varphi^2\partial\varphi\partial\varphi] + \dots + \sqrt{16\pi G} \varphi_{\mu\nu} T^{\mu\nu}, \quad (1)$$

where $T^{\mu\nu}$ is the energy-momentum tensor and $[\varphi^n\partial\varphi\partial\varphi]$ is a shorthand notation for a sum over the possible Lorentz invariant terms of the form $\varphi^n\partial\varphi\partial\varphi$. For example, $[\partial\varphi\partial\varphi]$ is explicitly given by the Fierz-Pauli Lagrangian [18], the first order linear approximation of GR that leads to Newton's gravity in the case of static ($v \ll c$) bodies:

$$[\partial\varphi\partial\varphi] = \frac{1}{2}\partial^\lambda\varphi_{\mu\nu}\partial_\lambda\varphi^{\mu\nu} - \frac{1}{2}\partial^\lambda\varphi_\mu^\mu\partial_\lambda\varphi_\nu^\nu - \partial^\lambda\varphi_{\lambda\nu}\partial_\mu\varphi^{\mu\nu} - \partial^\nu\varphi_\lambda^\lambda\partial^\mu\varphi_{\mu\nu}. \quad (2)$$

Although the coupling $\sqrt{16\pi G}$ is small, the terms $(16\pi G)^{n/2}[\varphi^n\partial\varphi\partial\varphi]$ may become important for large enough $\varphi_{\mu\nu}$, i.e. massive enough bodies. We will first explore the phenomenology of Eq. (1) and then discuss how to quantitatively study its consequences, following a numerical method developed In Ref. [14].

The polynomial expansion Eq. (1) allows to interpret \mathcal{L}_{GR} in the language of particle physics. The first term $\mathcal{L}_{GR}^1 \equiv [\partial\varphi\partial\varphi]$ generates the free graviton propagator, producing in the static limit Newton's law with its familiar $1/r^2$ dependence. Higher order terms $\mathcal{L}_{GR}^n \equiv (16\pi G)^{n/2}[\varphi^n\partial\varphi\partial\varphi]$ represent graviton vertices with $n+2$ external legs: the gravitons interact with each other. For a massive enough static two-body system, the gravitons are preferably attracted toward the region of highest graviton density, i.e. the line joining the two bodies. This and the fact that the two bodies are static render the two dimensions transverse to this line irrelevant. The system is reduced to one dimension. In one dimension, a force mediated by massless carriers is constant. Thus, the attraction between the two bodies becomes constant rather than varying as $1/r^2$ as in the free-propagation case. The binding is stronger, effectively leading to an increase of the system mass. This increase contributes to a dark mass. For a homogeneous continuous disk distribution, the gravitons are attracted to the disk plane. In the extreme case, the propagation of gravitons is confined in two dimensions, resulting in a $1/r$ dependence of the gravity of the disk acting on a mass within the disk. For a spherically symmetric system there is no preferred direction(s) and the force from the sphere acting on one of its constituents varies as $1/r^2$. Thus, the mutual interaction of gravitons and the symmetry of the matter distribution could explain the correlation between M/L and ε . Namely, the interaction, or the total effective mass of the system, varies from the familiar $1/r^2$ law for $\varepsilon = 0$ systems to a $1/r$ law for $\varepsilon = 1$ systems. The question is whether elliptical galaxies are massive enough to make the terms \mathcal{L}_{GR}^n , $n > 1$, non-negligible. We now discuss quantitatively this question.

For the static case of two point-like bodies located at r_1 and r_2 , the tensors in Eq. (1) can be approximated by their time-time components [14] and \mathcal{L}_{GR} becomes:

$$\mathcal{L}_{GR} = \sum_{n=0}^{\infty} a_n (16\pi G)^{n/2} \varphi^n \partial\varphi\partial\varphi + \sqrt{16\pi G} \varphi \left(\delta^{(4)}(r - r_1) + \delta^{(4)}(r - r_2) \right), \quad (3)$$

where $\varphi \equiv \varphi_{00}$ and a_n are coefficients comparable to unity. For order of magnitude estimates, we can take $a_n = 1$ for all n . (One can show that $a_1 = 1$ by deriving the potential by using Eq. (3) for weak fields and comparing it to the Einstein–Infeld–Hoffmann equations [19].) In the static case, the potential is given by the two-point Green function $G_{2p}(r)$. In the Feynman path-integral formalism it is:

$$G_{2p}(r_1 - r_2) = \frac{1}{Z} \int D\varphi \varphi(r_1) \varphi(r_2) e^{-iS}, \quad (4)$$

where $S \equiv \int d^4x \mathcal{L}_{GR}$ is the action, $\int D\varphi$ sums over all possible field configurations, and $Z = \int D\varphi e^{-iS}$. In the static case, the time dimension can be ignored. The gravity field φ can then be numerically calculated at each site of a 3D lattice by using the standard Metropolis Monte-Carlo method. The method can be tested for known cases. Ignoring the terms \mathcal{L}_{GR}^n , $n > 1$, leads to the expected Newtonian potential, $G_{2p}(r) \propto 1/r$. Another check can be done by adding a fictitious graviton mass term in the Lagrangian, $m^2\varphi^2$. This leads to the expected Yukawa potential, $G_{2p}(r) \propto e^{-mr}/r$. These results are shown in Fig. 3. Including the terms \mathcal{L}_{GR}^n with $1 \leq n \leq 2$ yields roughly linear potentials in the case of a system of two bodies of typical galaxy mass, $M \sim 10^{12} M_\odot$, see Fig. 4. These calculations indicate that galaxies are in the regime in which the terms \mathcal{L}_{GR}^n , $n > 1$, become important. Symmetry arguments imply that the effect decreases when the system becomes more and more spherically symmetric. Consequently, the total effective mass of galaxies should correlate with its ellipticity, with little dark matter in the roundest systems. The symmetry arguments also imply that for disks with baryonic mass densities decreasing exponentially with radius, as for disk galaxies, rotation curves should reach a plateau [14], a well known manifestation of dark matter [2]. The calculations in Ref. [14] also agree well with galaxy cluster dynamics and the Bullet Cluster observations [20].

What can QCD teach us about dark matter? Gravity is not the only force that involves self-interacting fields. The theory of the strong interaction of quarks and gluons, QCD, is the archetypical self-interacting field theory. It is consequently worthwhile to explore the parallels between gravity and QCD. Both are Yang-Mills (non-Abelian) theories for which the symmetry group is non-commutative. The underlying reason is that the gauge charges in

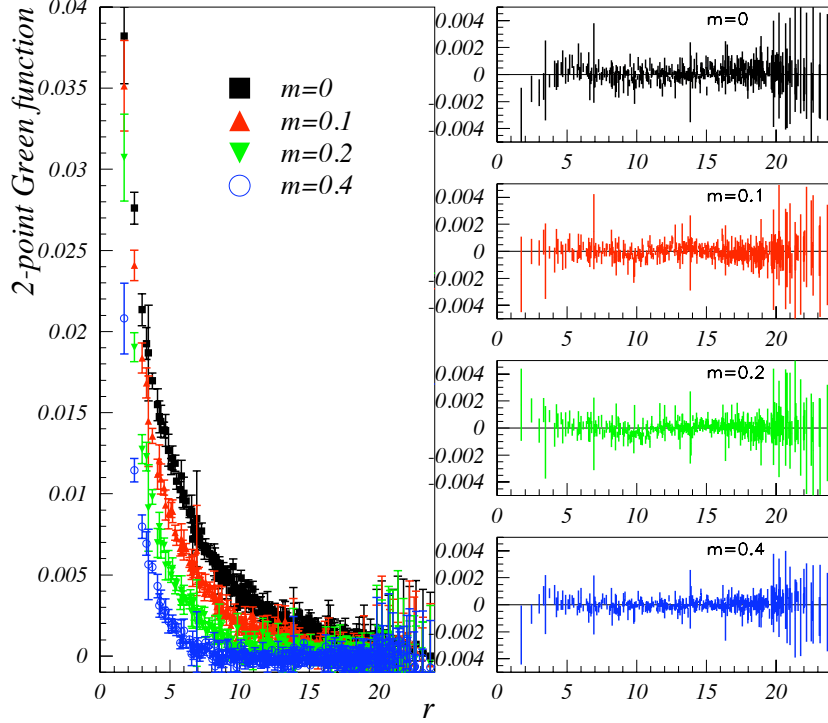


FIG. 3: Potentials obtained by Monte-Carlo calculations of the gravitational field when the higher order terms $\mathcal{L}_{GR}^n \equiv (16\pi G)^{n/2} [\varphi^n \partial \varphi \partial \varphi]$ in \mathcal{L}_{GR} are ignored and a fictitious mass term $m^2 \varphi^2$ is added (left). Newton's case corresponds to $m = 0$. Yukawa potentials are obtained when $m \neq 0$. Residuals from the e^{-mr}/r expectation are shown on the right.

both theories (the color charges for QCD and the energy-momentum tensor for gravity) are matrices, and thus non-commuting quantities. The physical consequence is that the force carriers (gluons for QCD and gravitons for gravity) mutually interact. Phenomenologically, the interactions occur because gluons carry color charges and gravitons have energy-momentum. Mathematically, the field Lagrangians for QCD and gravity have a similar form, although with different Lorentz structures: gluons have spin 1, so QCD is a vector field; gravitons have spin 2, so gravity is a tensor field. The QCD Lagrangian, without the matter term, is:

$$\mathcal{L}_{QCD} = \psi_{\mu\nu}^a \psi_a^{\mu\nu}, \quad (5)$$

with $\psi_{\mu\nu}^a = \partial_\mu \psi_\nu^a - \partial_\nu \psi_\mu^a - \sqrt{4\pi\alpha_s} f_{abc} \psi_\mu^b \psi_\nu^c$, where ψ_μ^a are the gluon fields, a the gluon color indices and f_{abc} the SU(3) structure constants. Expanding \mathcal{L}_{QCD} leads to

$$\mathcal{L}_{QCD} = [\partial\psi\partial\psi] + \sqrt{4\pi\alpha_s}[\psi^2\partial\psi] + 4\pi\alpha_s[\psi^4], \quad (6)$$

with the explicit structure $[\partial\psi\partial\psi] = 2\partial_\mu \psi_\nu^a \partial^\mu \psi_a^\nu - 2\partial_\mu \psi_\nu^a \partial^\nu \psi_a^\mu$, $[\psi^2\partial\psi] = 2f_{abc} [(\partial_\mu \psi_\nu^a) \psi_b^\mu \psi_c^\nu - (\partial_\mu \psi_\nu^a) \psi_b^\nu \psi_c^\mu]$ and $[\psi^4] = f_{abc} f_{ade} \psi_\mu^b \psi_\nu^c \psi_d^\mu \psi_e^\nu$. This is to be compared to the first three terms of \mathcal{L}_{GR} in Eq. (1). The comparison reveals the similarity in the structure of \mathcal{L}_{QCD} and \mathcal{L}_{GR} . There are, however, significant differences. One is that \sqrt{G} is small while α_s is large for distances typical of hadron sizes ($\sim 10^{-16}$ m). Another difference is that the gravity field is spin-2 and hence always attracts, while the strong force field is spin-1 and can either attract or repulse. That \sqrt{G} is small can be compensated by the fact that gravity always attracts. It is then reasonable to expect, for massive enough systems, that gravity's self-interaction should yield effects similar to those characterizing the hadron structure. Such effects in hadron structure phenomenology are:

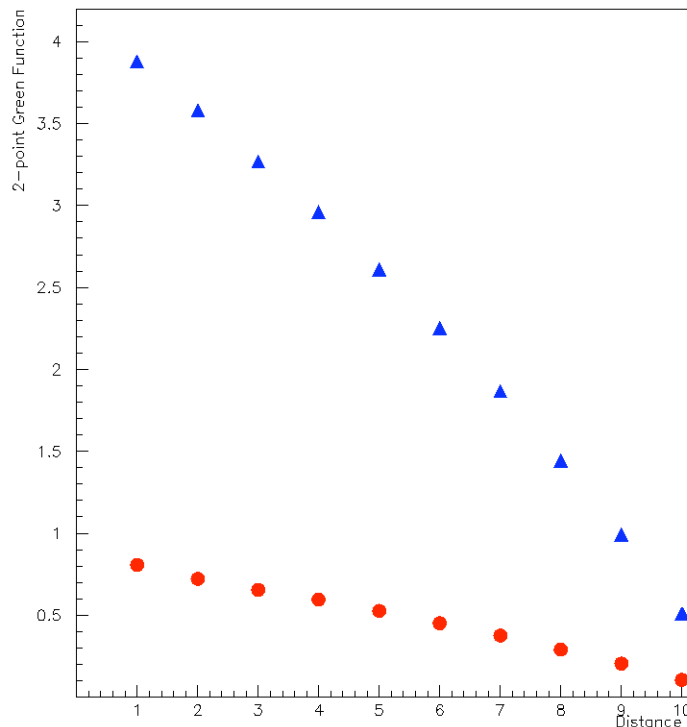


FIG. 4: Potentials obtained when the terms \mathcal{L}_{GR}^n , $n = 1, 2$, in \mathcal{L}_{GR} are included. The potentials are now linear. The red circles (blue triangles) are for a 2-body system, each body of mass $10^{12}M_{\odot}$ ($3.2 \times 10^{12}M_{\odot}$) typical of elliptical galaxies.

- The strength of QCD becomes large at long distances (quarks are confined). The accepted explanation (in the static case of heavy quarks) is that α_s is large and gluons are color-charged. The gluon flux between two quarks collapses into a flux tube inducing a string-like confining potential.
- For a family of hadrons, the square of the hadron mass m_H is proportional to the angular momentum J (“Regge trajectories”): $\log m_H = 0.5 \log J + c$, where c is a constant. The interpretation is that larger J imply larger centripetal forces and hence, to keep the quark system bound, larger string tension (i.e. binding energy): the more a hadron rotates, the larger its total mass.
- There are no strong interactions outside hadrons, except for small residual effects such as the force resulting from light hadron exchange (Yukawa forces). This is because gluons, as color-charged particles, are confined into hadrons as well.

We can confront this list to the following cosmological phenomena:

- The strength of gravity in galaxies or more massive systems is larger than expected, based on the observed amount of visible matter. The accepted explanation of the increase in gravity’s strength is the existence of additional, non-baryonic, matter.
- The luminosities of disk galaxies or, equivalently, their luminous masses M_* , are related to their maximum rotation speed v (Tully-Fisher relation): $\log M_* = 3.9 \log v + 1.5$. The faster a galaxy rotates, the larger its total mass.
- Dark energy effectively acts as negative pressure, currently balancing the effect of matter’s attraction on the expansion of the universe.

There is an intriguing correspondence between the two lists. It is tempting to attribute it to the similarity between the Lagrangians of gravity and QCD. Such origin of dark matter and dark energy would yield natural explanations of the dark matter-baryon coincidence [21] and the cosmic coincidence problems [22]. It would also explain the negative results for experimental searches of WIMP [23] and axion dark matter candidates [24].

Summary We discussed a strong correlation between the dark matter content of elliptical galaxies and their ellipticities. It implies that the roundest elliptical galaxies contain little dark matter, a puzzling fact in the context of galaxy formation and cold dark matter models. The mutual interaction of gravitons suggests an explanation. It also directly explains the flat rotation curves of disk galaxies and cluster dynamics. Such observations can be paralleled with QCD phenomenology. The similar forms of the field Lagrangians of gravity and QCD may explain the observed correspondences.

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